التحليل الاحتمالي لنظام مكون من وحدتين إحداهما موصلة والأخرى احتياط (معرضين لنوعين من الخطأ) غير متماثلتين وإضافة رجل التفتيش للنظام

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الملخص العربي :

تتناول هذه الورقة تأثير إضافة رجل التفتيش والصيانة خارج نظام يحتوي على وحدتين غير متماثلتين . كل منهما لها نوعان من الخطأ(خطأ من النوع الأول- خط من النوع الثاني) . وباستخدام خواص ماركوف حصلنا على دالة الاعتماد ومتوسط العمر في حالة الاستقرار. علما بان النظام يتوقف عن العمل عند فشل كلا الوحدتين ومعدل الفشل يتبع التوزيع الأسي لمعامل الفشل وتمكنا من الحصول على دالة الاعتماد ومتوسط العمرو توقع عدد مرات زيارات رجل التفتيش ونتائج هذه الورقة مدعمة عدديا وبيانيا باستخدام برامج الحاسب الألي والنتائج التي توصلت إليها توضح مدي تأثير إضافة رجل التفتيش والصيانة للنظام .

STATISTICAL ANALYSIS OF A TWO-UNT COLD STANDBY SYSTEM WITH TWO-TYPES OF FAILURE AND PREVENTIVE MAINTENANCE

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Abstract: Problem statement: This study presents the statistical analysis of two dissimilar parallel units in cold standby. Each unit works in two different types of failure as failure of type I and failure of type II. System fails when both

units fail totally. The system goes for preventive maintenance at random epochs. The failure and repair time and preventive maintenance time are assumed to have different arbitrary distribution .

Introduction

Many authors as [1,2,3and 4]have studied the two unit redundant systems with one type of failure . The purpose of the present paper is deal with the statistical analysis of a two dissimilar units redundant systems with two types of failure. Initially one unit is operative and the other is kept as cold standby, i.e.it does not fail while standing by. Each unit works in two different types of failures. The system fails when both units fail totally. The system goes for preventive maintenance at random epochs. The failure and repair time and preventive maintenance time are assumed to have different arbitrary distribution .

The following system characteristic:-

i. Mean time to system failure.

The following assumptions are adopted for the system:-

1. The system consists of two dissimilar parallel units. Initially one unit is operative and the other unit is kept as cold standby.

2. Standby is switched to operative state in negligible time.

3. A repaired unit works as a good as new.

5. Each unit has two types of failure.

6. Preventive maintenance (e.g., overhaul, inspection, minor repairs, etc.) is provided to this system at random epochs when

the system is in the state S_0 ,where both the components are normal.

The following notations are adopted for the system:-

$F_i(t), f_i(t)$ cdf and pdf of time to failure of i^{th} unit in type I where, i=1, 2.

$G_i(t), g_i(t)$ cdf and pdf of time to repair of i^{th} unit in type I where, i=1, 2.

$x_i(t), X_I(t)$ cdf and pdf of time to failure of i^{th} unit in type II where, i=1, 2.

$y_i(t), Y_I(t)$ cdf and pdf of time to repair of i^{th} unit in type II where, i=1, 2.

E_0 state of the system at time t=0 (initial stat).

E_i set of all possible states of the system S_i ,I=0,1,2,......,10

$q_{ij}(t), Q_{ij}(t)$ pdf and cdf time for transition from state s_i to state s_j where

i,j =0,1,2,......,10.

μ_i mean sojourn time in state S_i, $\mu_i = \sum \mu_{ij}$.

P_{ij} the transition probability from state s_i to state s_j .

$\pi_i(t)$ cdf of the time to system failure when the starting state $E_0 = S_i \in E_i$.

$A_i(t)$	p [system is available at epoch $t/E_0 = S_i \in E_i$].
$\nu(t)$	pdf of time for taking a unit into **PM**, $\nu(t) = n\exp(-nt), n, t \succ 0$,
$u(t)$	pdf of **PM** time, $u(t) = w\exp(-wt), w, t \succ 0$.

Stochastic Behavior of System

Figure (1) shows the state of the system

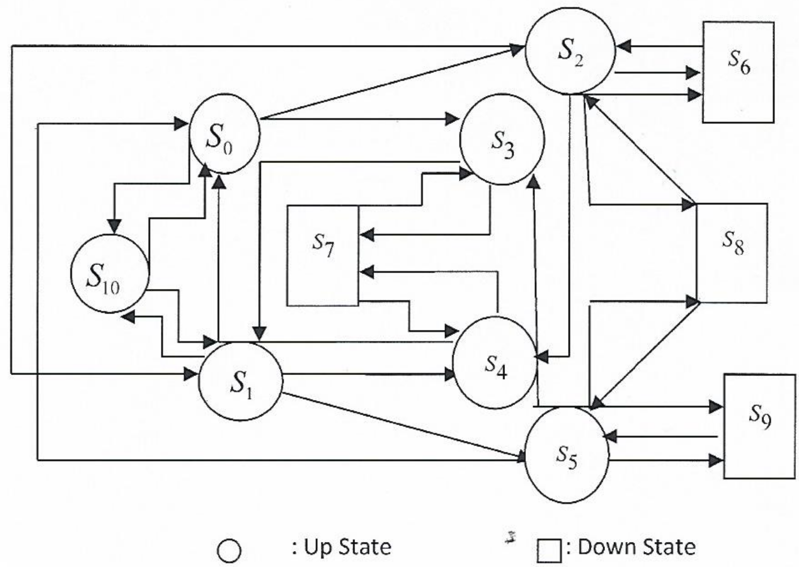

Figure 1: State transition diagram

The system can take one of the following states:-

i. $S_0(O_1, ST_2)$:The first unit is operative and the second unit is kept as cold standby.

ii. $S_1(ST_1, O_2)$:The first unit kept as cold standby and the second unit is operative.

iii. $S_2(Fr1, o2)$: The first unit is total failure of type I after operative and the second unit is operative after standby.

iv. $S_3(Fr2_1, O_2)$:The first unit is total failure of type II after operative and the second unit is operative after standby.

v. $S_4(O_1, F_{r1_2})$: The first unit is operative after standby and the second unit is total failure of type I after operative.

vi. $S_5(O_1, F_{r2_2})$:The first unit is operative after standby and the second unit is total failure of type II after operative.

vii. $S_6(F_{r1_1}, F_{r1_2})$:The two units are total failure of type I.

viii. $S_7(F_{r2_1}, F_{r1_2})$:The first unit is total failure of type II and the second unit is total failure of type I.

ix. $S_8(F_{r1_1}, F_{r2_2})$:The first unit is total failure of type I and the second unit is total failure of type II.

x. $S_9(F_{r2_1}, F_{r2_2})$:The two units are total failure of type II.

xi. $S_{10}(O_N, ST_N)$:The two units are under preventive maintenance.

Where, O_i The i^{th} unit is operative,

ST_i The i^{th} unit is standby,

F_{r1_i} The i^{th} unit is failure of type I,

F_{r2_i} The i^{th} unit is failure of type II.

Transition Probabilities and Mean Sojourn Times

Let $P_{ij} = Q_{ij}^{*}(0)$ be the one step transition probability from state S_i to state S_j

where, i , j=0,1,2,...........,10.

1. In state S_0 ,there are two transition can be considered one to state S_2 and the other to state S_3 and the other to state S_{10}.Therefore,

$$P_{01}=\int_0^\infty f_1(t)\overline{X_1}(t)dt,\qquad P_{03}=\int_0^\infty x_1\overline{F}(t)dt\ ,\qquad P_{0,10}=\int_0^\infty\ dt\ .$$

Thus,

$$P_{01}+P_{03}+P_{010}=1$$

2. Similarly, in state S_1 ,there are two transitions can be considered one to state S_4 and the other to state S_5 . Therefore,

$$P_{14}=\int_0^\infty f_2(t)\overline{X_2}(t)dt\ ,\qquad P_{15}=\int_0^\infty x_2(t)\overline{F_1}(t)dt\ ,$$

$$P_{1,10}=\int_0^\infty\ dt$$

Thus,

$$P_{14}+P_{15}=1$$

3.In state S_2 there are three transitions can be considered one to state S_1 ,the second to state S_6 and the third to state S_8 .Therefore,

$$P_{21} = \int_0^\infty g_1(t)\overline{F_2}(t)\overline{X_2}(t)dt\,, \qquad P_{26} = \int_0^\infty f_2(t)\overline{G}(t)\overline{X_2}(t)dt\,,$$

$$P_{28} = \int_0^\infty x_2(t)\overline{F_2}(t)\overline{G_1}(t)dt\,.$$

Thus,

$$P_{21} + P_{26} + P_{28} = 1.$$

4. Similarly, in state S_3 there are three transitions can be considered one to state S_1 ,the second to state S_7 and the third to state S_9 .Therefore,

$$P_{31} = \int_0^\infty y_1(t)\overline{F_2}(t)\overline{X_2}(t)dt\,, \qquad P_{37} = \int_0^\infty f_2(t)\overline{Y_1}(t)\overline{X_2}(t)dt\,,$$

$$P_{39} = \int_0^\infty x_2(t)\overline{Y_1}(t)\overline{F_2}(t)dt\,.$$

Thus,

$$P_{31} + P_{37} + P_{39} = 1.$$

5. Similarly, in state S_4 there are three transitions can be considered one to state S_0 ,the second to state S_6 and the third to state S_7. Therefore,

$$P_{40} = \int_0^\infty g_2(t)\overline{F_1}(t)\overline{X_1}(t)dt\,, \qquad P_{46} = \int_0^\infty f_1(t)\overline{Y_2}(t)\overline{X_1}(t)dt\,,$$

$$P_{47} = \int_0^\infty x_1(t)\overline{F_1}(t)\overline{G_2}(t)dt$$

Thus,

$$P_{40} + P_{46} + P_{47} = 1.$$

6.Similarly, in state S_5 there are three transitions can be considered one to state S_0 ,the second to state S_8 and the third to state S_9 .Therefore,

$$P_{50} = \int_0^\infty y_2(t)\overline{F_1}(t)\overline{X_1}(t)dt\,, \qquad P_{58} = \int_0^\infty f_1(t)\overline{Y_2}(t)\overline{X_1}(t)dt\,,$$

$$P_{59} = \int_0^\infty x_1(t)\overline{F_1}(t)\overline{Y_2}(t)dt$$

Thus,

$$P_{50} + P_{58} + P_{59} = 1.$$

7.In state S_6 , there are two transitions can be considered one to state S_2 and the other to state S_4 .Therefore,

Thus

$$P_{62} = \int_0^\infty g_2(t)\overline{G_1}(t)dt\,, \qquad P_{64} = \int_0^\infty g_1(t)\overline{G_2}(t)dt$$

$$P_{62} + P_{64} = 1$$

8. Similarly, in state S_7 ,there are two transitions can be considered one to state S_3 and the other to state S .Therefore,

Thus,

$$P_{73} = \int_0^\infty g_2(t)\overline{Y_1}(t)dt\,, \qquad P_{74} = \int_0^\infty y_1(t)\overline{G_2}(t)dt$$

$$P_{73} + P_{74} = 1.$$

9. Similarly, in state S_8 , there are two transitions can be considered one to state S_2 and the other to state S_5 .Therefore,

$$P_{82} = \int_0^\infty y_2(t)\overline{G_1}(t)dt\,, \qquad P_{85} = \int_0^\infty g_1(t)\overline{Y_2}(t)dt$$

$$P_{82} + P_{85} = 1.$$

10. Similarly, in state S_9 , there are two transitions can be considered one to state S_3 and the other to state S_5 . Therefore,

$$P_{93} = \int_0^\infty y_2(t)\overline{Y_1}(t)dt\,, \qquad P_{95} = \int_0^\infty g_1(t)\overline{Y_2}(t)dt$$

$$P_{93} + P_{95} = 1.$$

11. Similarly, in state S_{10} , there are two transitions can be considered one to state S_0 . and to state S_1 Therefore,

$$P_{10,0} + P_{10,1} + = 1$$

The mean sojourn times , in state are given by

$$\mu_0 = \int_0^\infty \overline{F_1}(t)\overline{X_1}(t)dt\,, \qquad \mu_1 = \int_0^\infty \overline{F_2}(t)\overline{X_2}(t)dt\,,$$

$$\mu_3 = \int_0^\infty \overline{Y_1}(t)\overline{F_2}(t)\overline{X_2}(t)dt\,, \qquad \mu_4 = \int_0^\infty \overline{F_1}(t)\overline{X_1}(t)\overline{G_2}dt\,,$$

$$\mu_5 = \int_0^\infty \overline{Y_2}(t)\overline{F_1}(t)\overline{X_1}(t)dt\,,$$

$$\mu_6 = \int_0^\infty \overline{G_1}(t)\overline{G_2}(t)dt\,, \qquad \mu_7 = \int_0^\infty \overline{Y_1}(t)\overline{G_2}(t)dt\,, \quad \mu_8 = \int_0^\infty \overline{G_1}(t)\overline{Y_2}(t)dt\,,$$

$$\mu_9 = \int_0^\infty \overline{Y_1}(t)\overline{Y_2}(t)dt\,, \qquad \mu_{10} = \int_0^\infty \overline{F_2}(t)\overline{X_2}(t)dt\,. \qquad (1)$$

Mean Time to System Failure:

Time to system failure can be regarded as the first passage to any of the failed states $S_6 - S_9$ which is considered as absorbing. Employing the arguments used for regenerative process the following recursive relations for $\tilde{\pi}_i(t)$ are obtained when $E_0 = S_i$

$$\pi_0(t) = Q_{02}(t)(s)\pi_2(t) + Q_{03}(t)(s)\pi_3(t) + Q_{010}(t)(s)\pi_{10}(t)\,,$$

$$\pi_1(t) = Q_{14}(t)(s)\pi_4(t) + Q_{15}(t)(s)\pi_5(t) + Q_{1,10}(t)(s)\pi_{10}(t)\,,$$

$$\pi_2(t) = Q_{21}(t)(s)\pi_1(t) + Q_{26}(t) + Q_{28}(t)\,,$$

$$\pi_3(t) = Q_{31}(t)(s)\pi_1(t) + Q_{37}(t) + Q_{39}(t)\,,$$

$$\pi_4(t) = Q_{40}(t)(s)\pi_0(t) + Q_{46}(t) + Q_{47}(t),$$

$$\pi_5(t) = Q_{50}(t)(s)\pi_0(t) + Q_{58}(t) + Q_{59}(t),$$

$$\pi_{10}(t) = Q_{10,0}(t)(s)\pi_0(t) \ + Q_{10,1}(t)(s)\pi_1(t),$$

(2)

After using Laplace-stieltjes transform for equations (2) and solving for $\tilde{\pi}_0(0)$,we get. The Mean Time System Failure which is given by

$$MTSF = \frac{-d}{ds}\tilde{\pi}_0(s)|_{s=0} = \frac{|D'(0) - N'(0)|}{D(0)}, \qquad (3)$$

Where,

$$D(0) = 1 - (p_{10,1}p_{1,10} + (p_{02}p_{21} + p_{03}p_{31})(p_{1,10}p_{10,0} + p_{14}p_{40} + p_{15}p_{50})$$
$$+ p_{0,10}(p_{10,0} + p_{10,1}(p_{14}p_{40} + p_{15}p_{50}))),$$

$$|D'(0) - N'(0)| = \begin{array}{l}(\mu_0 p_{21} + \mu_2 p_{02} + \mu_3 p_{03}B)A + (\mu_4 p_{14} + \mu_5 p_{15})B + \\ \mu_{10}(p_{0,10} + p_{1,10}(p_{02}p_{21} + p_{03}p_{31})),\end{array}$$

$$A = p_{1,10}p_{10,0} + p_{14}p_{40} + p_{15}p_{50},$$

$$B = p_{0,10}p_{10,1} + p_{02}p_{21} + p_{03}p_{31}.$$

Graphical Representation:

in Equation (3) we get Table 1.

Table 1:Relation Between $p_{10,0}, p_{0,10}$ an the MTSF

$p_{10,0}$	MTSF	$p_{0,10}$	MTSF
0.9	0.7814	0.9	0.781357
0.8	0.7718	0.8	0.80081
0.7	0.7615	0.7	0.821135
0.6	0.7501	0.6	0.842391
0.5	0.7376	0.5	0.864645
0.4	0.7237	0.4	0.887967
0.3	0.7083	0.3	0.912437
0.2	0.7237	0.2	0.938142
0.1	0.70845	0.1	0.965177
0.09	0.69116	0.09	0.967958
0.08	0.67182	0.08	0.970753

CONCLUSION:

Table 1: show that the present additional preventive maintenance lead to improve the values of the mean time to system failure are decreases by using preventive maintenance as shown from their behaviors when plotted against p_{21} or p_{31} or p_{02}.

Figures (2,3 and 4) demonstrate the following results which are only to expected.

As both the transition probabilities p_{21}, p_{31} and p_{02} increases:

1-the mean time to system failure with PM decreases.

MSTF

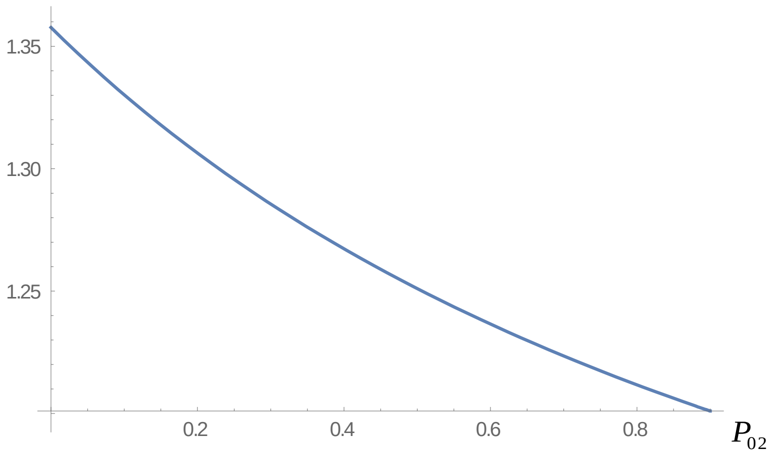

Figure 2:represent relation between $0 \le p_{02} \le 0.9)$ and MTSF

MSTF

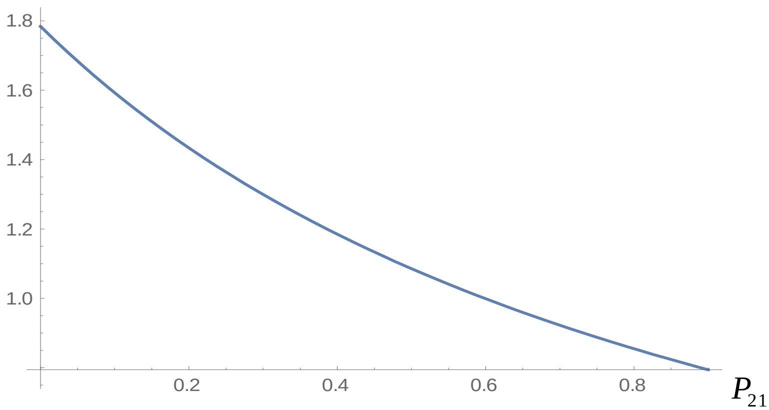

Figure 3:represent relation between $(0 \le p_{21} \le 0.9)$ and MTSF

MSTF

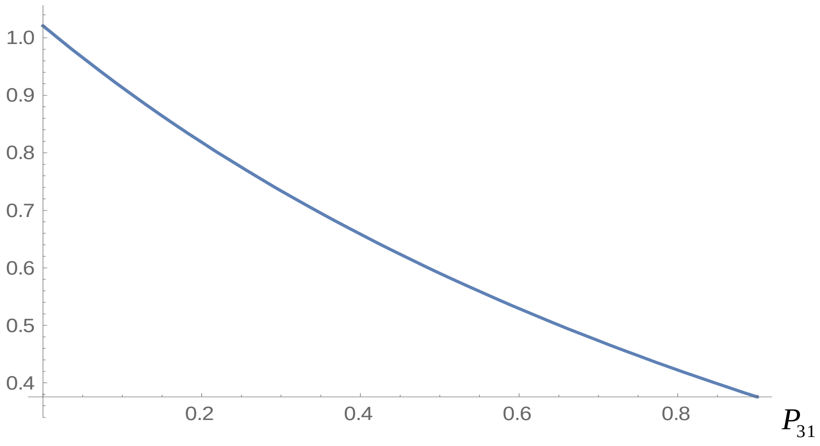

Figure 4:represent relation between $(0 \le p_{31} \le 0.9)$ and MTSF

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