تمثيل موجة الظل لحل صدمة دلتا المنقسمة للشكل العام من مسألة ريمان

د. دلال عبد السلام المبروك ضو - كلية العلوم – العجيلات- جامعة الزاوية

الملخص :

يتم إجراء صدمة دلتا المنقسمة لتوفير طريقة لاستخدامها في المعادلات التفاضلية الجزئية غير الخطية. صدمة دلتا المنقسمة تمثيل لدالة (ديراك) دلتا. لم نستخدم الحل الكلاسيكي. قمنا ببناء حل باستخدام أسلوب موجة الظل [3] مع صدمة دلتا المنقسمة للشكل العام من مسألة ريمان.

Shadow wave representation of split delta shock solution for a general form of Riemann problem

Dalal Abdulsalam Elmabruk Daw

Department of Mathematics- Faculty of Science\ Ajelat-University of Zawia

Abstract

A split delta shock is made to provide a way of using it in nonlinear partial differential equations. It is a representation of the Dirac delta function. We do not use classical solution. We construct a solution using shadow wave approach [3] with a split delta shock for a general form of Riemann problem.

1-Introduction

The aim of this paper is to solve the following general form of Riemann problem without classical solutions

$$
\begin{aligned}
&u_t + \left(\frac{a_0 + a_1 u}{v} + \frac{b_0 + b_1 v}{u}\right)_x = 0, \qquad u(x,0) = \begin{cases} u_0, & x < 0 \\ u_1, & x > 0 \end{cases} \\
&v_t + \left(\frac{\bar{a}_0 + \bar{a}_1 u}{v} + \frac{\bar{b}_0 + \bar{b}_1 v}{u}\right)_x = 0, \qquad v(x,0) = \begin{cases} v_0, & x < 0 \\ v_1, & x > 0. \end{cases}
\end{aligned} \tag{1}
$$

where $u, v \in \Omega$, $\Omega \subset R^2$ is a physical domain, i.e. a set of all possible values for *(u, v)* and $\bar{a}_0, \bar{a}_1, \bar{b}_0, \bar{b}_1$ denotes to the closure of a_0, a_1, b_0, b_1, respectively. Physical domain for solutions is given by $u > 0, v > 0$. By using the procedure for split delta shock ([1], [5]) as a part of a solution . A split delta shocks are introduced in order to solve some systems of conservation laws without classical; solutions (see [1]). The shadow waves are defined by nets of piecewise constant functions for time variable t fixed parameterized by some small parameter ε > 0 and bounded in L^1_{loc}(R). A use of such parameter enable us to include the split delta function as a part of solution. The main idea is to using the most usual admissibility condition is to be over compressive, i.e. All that split delta shocks are required characteristics should run into the shock curve, with shadow wave (SDW for short) (see [3]). Another admissible solution is delta shock with a constant strength that propagates along characteristic. It is called a delta contact discontinuity (see [1] or [8]). That is possible for systems having a linearly degenerate field. In general, we get a values for a speed c and a locus L((u_0, v_0)) being a curve.

2- A construction of SDW solution with split delta shock

Suppose that there exist a two components split delta shock solution.

$$u(x,t) = \begin{cases} u_0\,, & x \le ct \\ u_1\,, & x \ge ct \end{cases} + (\alpha_0\delta^- + \alpha_1\delta^+)t\,, v(x,t) = \begin{cases} v_0\,, & x \le ct \\ v_1\,, & x \ge ct \end{cases} + (\beta_0\delta^- + \beta_1\delta^+)t \qquad (2),$$

defined in [10]and[11] to some conservation law system linear in one of solution component, v for definiteness. In [12] one can look about the definition of split delta shocks. The values $(\alpha_0+\alpha_1)t$ and $(\beta_0+\beta_1)t$ are called strength of a split delta shock. For a given point (u_0, v_0) in a physical domain $\Omega \subset R^2$ for (1), a set of all (u_1,v_1) in the domain such that there exists a split delta shock connecting these states is called split delta locus denoted by L((u_0,v_0)).

A construction of a convenient SDW solutions (see [3]) is straightforward, by putting

$$u_{1,\varepsilon} = u, \quad u_{2,\varepsilon} = v, \quad \lim_{\varepsilon\to 0} a_\varepsilon u_{1,\varepsilon} = \propto_0, \quad \lim_{\varepsilon\to 0} b_\varepsilon u_{2,\varepsilon} = \propto_1$$

also

$$\lim_{\varepsilon\to 0} a_\varepsilon v_{1,\varepsilon} = \beta_0 , \qquad \lim_{\varepsilon\to 0} b_\varepsilon v_{2,\varepsilon} = \beta_1$$

3- The overcompressibility condition of the SDW solution with split delta shock

In this part we can try to substitute the SDW solution with using the procedure for split delta shock (see [3]) to the general form of Riemann problem (1). To satisfy the admissibility criteria for SDWs we will use the overcompressibility condition.

The *simple SDW* given by the following formula

$$u(x,t) = \begin{cases} u_0 \ , & x < (c - a_\varepsilon)t \\ u_{1,\varepsilon} \ , & (c - a_\varepsilon)t < x < ct \\ u_{2,\varepsilon} \ , & ct < x < (c + b_\varepsilon)t \\ u_1 \ , & x > (c + b_\varepsilon)t \end{cases} \tag{3}$$

while a_ε, b_ε are smooth functions equal zero at t = 0 with growth order less or equal to ε.

Definition: The SDW of the form (3) is called overcompressive condition if

$$\lambda_1(u_0 , v_0) \geq c \geq \lambda_2 (u_1 , v_1)$$

i .e. all characteristics should run into curve . One can look in [9] or [10] for a detailed explanation of that admissibility condition.

Now we can construct SDW solution with split delta shocks in the following theorem.

Theorem1: *The general form of the Riemann problem (1) has a unique solution in the region where u and v are non- negative. The solution satisfies SDW solution with split delta shock of the form of the simple SDW (3).*

Proof:

We can try to substitute a simple shadow waves solution (3) in both equations of the system (1), we will use the procedure for split delta shock

For the first equation of (1) we have

$$I_1 \approx -\int_0^{\infty}\int_{-\infty}^{(c-a_\varepsilon)t}\Big((u_0\partial t\,\varphi(x,t) + (\frac{a_0+a_1u_0}{v_0} + \frac{b_0+b_1v_0}{u_0})\partial_x\varphi(\partial_x(x,t)\Big)\,dx\,dt$$

$$-\int_0^{\infty}\int_{(c-a_\varepsilon)t}^{ct}(u_{1,\varepsilon}\partial_t\ \varphi(x,t) + \frac{a_0+a_1u_{1,\varepsilon}}{v_{1,\varepsilon}} + \frac{b_0+b_1v_{1,\varepsilon}}{u_{1,\varepsilon}})\partial_x\varphi(\,x,t)dx\,dt$$

$$-\int_0^{\infty}\int_{ct}^{(c+b_\varepsilon)t} u_{2,\varepsilon}\partial_t\varphi(x,t) + \left(\frac{a_0+a_1u_{2,\varepsilon}}{v_{2,\varepsilon}} + \frac{b_0+b_1v_{2,\varepsilon}}{u_{2,\varepsilon}}\right)\partial_x\varphi(x,t)dx\,dt$$

$$-\int_0^{\infty}\int_{(c+b_\varepsilon)t}^{\infty}\left(u_1\partial_1\varphi(x,t) + \left(\frac{a_0+a_1u_1}{v_1} + \frac{b_0+b_1v_1}{u_1}\right)\right)\partial_x\varphi(x,t)dx\,dt$$

where $a_\varepsilon, b_\varepsilon \sim \varepsilon$

After integration by parts and calculation, we get

$$I_1 \approx \int_0^{\infty} u_0(c-a_\varepsilon)\varphi\big((c-a_\varepsilon)t,t\big)\,dt - \int_0^{\infty}\left(\frac{a_0+a_1u_0}{v_0} + \frac{b_0+b_1v_0}{u_0}\right)\varphi\big((c-a_\varepsilon)t,t\big)dt$$

$$+\int_0^{\infty} u_{1,\varepsilon}\varphi(c-a_\varepsilon)\,t,t)\ (a_\varepsilon)dt - \int_0^{\infty} u_{1,\varepsilon}\varphi(x,0)dx$$

$$-\int_0^{\infty}\left(\frac{a_0+a_1u_{1,\varepsilon}}{v_{1,\varepsilon}} + \frac{b_0+b_1v_{1,\varepsilon}}{u_{1,\varepsilon}}\right)(a_\varepsilon)\varphi\,\big((c-a_\varepsilon)t,t\big)dt$$

$$+\int_0^{\infty} u_{2,\varepsilon}(b_\varepsilon)\varphi((c+b_\varepsilon)t,t)\,dt + \int_0^{\infty} u_{2,\varepsilon}\varphi(x,0)dx$$

$$-\int_0^{\infty}\left(\frac{a_0+a_1u_{2,\varepsilon}}{v_{2,\varepsilon}} + \frac{b_0+b_1v_{2,\varepsilon}}{u_{2,\varepsilon}}\right)b_\varepsilon\,\varphi\,\big((c+b_\varepsilon)t,t\big)dt$$

$$-\int_0^{\infty} u_1(c+b_\varepsilon)\varphi((c+b_\varepsilon)t,t)\,dt + \int_0^{\infty}\left(\frac{a_0+a_1u_1}{v_1} + \frac{b_0+b_1v_1}{u_1}\right)\big((\varphi(c+b_\varepsilon)t,t\big)dt$$

The sign ,, ≈,, means a convergence to zero as $\varepsilon \to 0$.

Note that

$\int_{-\infty}^{0} u_0\varphi(x,0)dx \int_0^{\infty} u_1\varphi(x,0)dx =< u|_{t=0}, \varphi >$

and

$\int_{-\infty}^{0} u_{1,\varepsilon}\varphi(x,0)dx \int_0^{\infty} u_{2,\varepsilon}\varphi(x,0)dx =< u|_{t=0}, \varphi >$

that cancels with initial data and we will drop it in the rest of calculations. and we used the fact

$\varphi((c \pm a_\varepsilon)t,t) = \varphi(c,t) \pm \varphi\partial_t(c,t)a_\varepsilon t + O(\varepsilon^2)$

also

$\varphi((c \pm b_\varepsilon)t,t) = \varphi(c,t) \pm \varphi\partial_t(c,t)b_\varepsilon t + O(\varepsilon^2)$

where $u_{1,\varepsilon},\ u_{2,\varepsilon} \sim \frac{1}{\varepsilon}$

Since we using a split delta shock, then we put

$\lim_{\varepsilon\to 0} a_\varepsilon u_{1,\varepsilon} = \propto_0$, $\lim_{\varepsilon\to 0} b_\varepsilon u_{2,\varepsilon} = \propto_1$

also $\lim_{\varepsilon\to 0} a_\varepsilon v_{1,\varepsilon} = \beta_0$, $\lim_{\varepsilon\to 0} b_\varepsilon v_{2,\varepsilon} = \beta_1$

Then we get the following equation

$$
\begin{aligned}
I_1 \approx -\int_0^{\infty} \left(c[u] - \left[\frac{a_0 + a_1 u}{v} + \frac{b_0 + b_1 v}{u}\right] + \left(a_\varepsilon u_{1,\varepsilon} + b_\varepsilon u_{2,\varepsilon}\right)\varphi(ct,t) \right) dt \\
- \int_0^{\infty} c\Big(\big(a_\varepsilon v_{1,\varepsilon} + b_\varepsilon v_{2,\varepsilon} \big) + \Bigg(\frac{a_0 + a_1 u_{1,\varepsilon}}{\boldsymbol{v_{1,\varepsilon}}} \\
+ \frac{b_0 + b_1 v_{1,\varepsilon}}{u_{1,\varepsilon}}\Bigg) a_\varepsilon \\
+ \left(\frac{a_0 + a_1 u_{2,\varepsilon}}{v_{2,\varepsilon}} + \frac{b_0 + b_1 v_{2,\varepsilon}}{u_{2,\varepsilon}}\right) b_\varepsilon \Big) t\ \partial_x\, \varphi(ct,t) dt \\
\approx 0 \qquad\qquad (4)
\end{aligned}
$$

Here we will name the following part of above equation (4) by (A)

$$\left(\frac{a_0+a_1 u_{1,\varepsilon}}{v_{1,\varepsilon}}+\frac{b_0+b_1 v_{1,\varepsilon}}{u_{1,\varepsilon}}\right)a_\varepsilon+\left(\frac{a_0+a_1 u_{2,\varepsilon}}{v_{2,\varepsilon}}+\frac{b_0+b_1 v_{2,\varepsilon}}{u_{2,\varepsilon}}\right)b_\varepsilon \qquad (A)$$

With the same argument, and with substitution

$$u_t \to v_t\ ,\ \left(\frac{a_0+a_1 u}{v}+\frac{b_0+b_1 v}{u}\right)\to\left(\frac{\bar{a}_0+\bar{a}_1 u}{v}+\frac{\bar{b}_0+\bar{b}_1 v}{u}\right)$$

for the second equation

$$v_t+\left(\frac{\bar{a}_0+\bar{a}_1 u}{v}+\frac{\bar{b}_0+\bar{b}_1 v}{u}\right)_x=0, \qquad v(x,0)=\begin{cases}v_0, & x<0\\ v_1, & x>0.\end{cases}$$

we get the following equation

$$I_2\approx-\int_0^\infty\left(c[v]-\left[\frac{\bar{a}_0+\bar{a}_1 u}{v}+\frac{\bar{b}_0+\bar{b}_1 v}{u}\right]+(a_\varepsilon v_{1,\varepsilon}+b_\varepsilon v_{2,\varepsilon})a_\varepsilon\varphi(ct,t)\right)dt$$

$$-\int_0^\infty\left(c(a_\varepsilon v_{1,\varepsilon}+b_\varepsilon v_{2,\varepsilon})+\left(\frac{\bar{a}_0+\bar{a}_1 u_{1,\varepsilon}}{v_{1,\varepsilon}}+\frac{\bar{b}_0+\bar{b}_1 v_{1,\varepsilon}}{u_{1,\varepsilon}}\right)a_\varepsilon+\left(\frac{\bar{a}_0+\bar{a}_1 u_{2,\varepsilon}}{v_{2,\varepsilon}}+\frac{\bar{b}_0+\bar{b}_1 v_{2,\varepsilon}}{u_{2,\varepsilon}}\right)\right)b_\varepsilon\, t\partial_x\varphi(ct,t)dt\approx 0$$

(5)

.In part (A), if both $u_{i,\varepsilon}$, $v_{i,\varepsilon}\sim\frac{1}{\varepsilon}$, $i=1,2$, then $\propto_0+\propto_1=0$ since all terms in (A) tend to zero.

The part (A) is not zero if and only if $u_{i,\varepsilon}$, $v_{i,\varepsilon}$ tend to a constant.

For example , $u_{1,\varepsilon}\approx A_1$, $\frac{b_0+b_1 v_{1,\varepsilon}}{u_{1,\varepsilon}}a_\varepsilon\approx\frac{b_1}{A_1}\beta_0$

Then we have the following cases:

Case I: Only one of $u_{i,\varepsilon}$, $v_{i,\varepsilon}$, $i = 1,2$ $is\ not$ $\frac{1}{\varepsilon}$

Say that $u_{1,\varepsilon} \sim \frac{1}{\varepsilon}$, $u_{2,\varepsilon} \sim \frac{1}{\varepsilon}$, $v_{1,\varepsilon} \sim \frac{1}{\varepsilon}$, $v_{2,\varepsilon} \approx B_2$

(all other cases are the same). Then from the equation (4), we have

$$\propto_0 + \propto_1 = k_1,$$

That implies

$$\frac{a_0 + a_1 u_{1,\varepsilon}}{v_{1,\varepsilon}} a_\varepsilon \approx 0\,, \qquad \frac{b_0 + b_1 v_{1,\varepsilon}}{u_{1,\varepsilon}} a_\varepsilon \approx 0$$

$$\frac{a_0 + a_1 u_{2,\varepsilon}}{v_{2,\varepsilon}} b_\varepsilon \approx \frac{a_1}{B_2} \propto_1\,, \qquad \frac{b_0 + b_1 v_{2,\varepsilon}}{u_{2,\varepsilon}} b_\varepsilon \approx 0$$

Then

$$ck_1 + \frac{a_1}{B_2} \propto_1 = 0 \qquad (6) \Leftrightarrow c(\propto_0 + \propto_1) + \frac{a_1}{B_2} \propto_1 = 0$$

Now from the equation (5), we have

$$\beta_0 + \beta_1 = k_2, \qquad then\ \ \beta_0 = k_2,\ \ \beta_1 = 0$$

That implies

$$\frac{\bar{a}_0 + \bar{a}_1 u_{1,\varepsilon}}{v_{1.\varepsilon}} a_\varepsilon \approx 0\,, \quad \frac{\bar{b}_0 + \bar{b}_1 v_{1,\varepsilon}}{u_{1.\varepsilon}} a_\varepsilon \approx 0$$

$$\frac{\bar{a}_0 + \bar{a}_1 u_{2,\varepsilon}}{v_{2.\varepsilon}} b_\varepsilon \approx \frac{\bar{a}_1}{B_2} \propto_1\,, \quad \frac{\bar{b}_0 + \bar{b}_1 v_{2,\varepsilon}}{u_{2,\varepsilon}} b_\varepsilon \approx 0$$

then

$$ck_2 + \frac{\bar{a}_1}{B_2} \propto_1 = 0 \qquad (7) \Leftrightarrow c(\beta_0 + \beta_1) + \frac{\bar{a}_1}{B_2} \propto_1 = 0$$

where *u* and *v* are non-negative, $\propto_{0,1} \in [0,\ k_1]$, $\beta_{0,1} \in [0, k_2]$.

Thus from (6) and (7), we have

$$c=-\frac{a_1}{B_2}\frac{\propto_1}{k_1}=-\frac{\bar{a}_1}{B_2}\frac{\propto_1}{k_2}\quad \text{and SDW exists only if}\quad \frac{a_1}{\bar{a}_1}=\frac{k_1}{k_2}$$

The Speed c is uniquely determined from the equation

$$\left(c[u]-\left[\frac{a_0+a_1v}{u}+\frac{b_0+b_1u}{v}\right]\right)\bar{a}_1=\left(c[v]-\left[\frac{\bar{a}_0+\bar{a}_1v}{u}+\frac{\bar{b}_0+\bar{b}_{1u}}{v}\right]\right)a_1 \qquad (8)$$

- If $v_{1,\varepsilon}\approx \mathrm{B}_1$ and others are $\frac{1}{\varepsilon}$, then SDW exists only if $\frac{a_1}{\bar{a}_1}=\frac{k_1}{k_2}$, too .
- If $u_{1,\varepsilon}\approx \mathrm{A}_1$ and others are $\frac{1}{\varepsilon}$, then SDW exists only if $\frac{b_1}{\bar{b}_1}=\frac{k_1}{k_2}$.
- The same for $u_{2,\varepsilon}\approx A_2$ (c is different)

In all these cases c is uniquely determined as in (8).

Case II*:* let $u_{2,\varepsilon}\approx A_{2,}\ v_{2,\varepsilon}\approx B_2,\ u_{1,\varepsilon},\ v_{1,\varepsilon}\sim\frac{1}{\varepsilon}$

(the same for other side $u_{1,\varepsilon}\approx A_1,\ v_{1,\varepsilon}\approx B_1$)

From the equation (4) , we have $\propto_0=k_1$, and from equation (5), we have $\beta_0=\mathrm{k}_2$
Then all

$$\frac{a_0+a_1u_{1,\varepsilon}}{v_{1,\varepsilon}}a_\varepsilon\to 0,\quad \frac{b_0+b_1v_{1,\varepsilon}}{u_{1,\varepsilon}}a_\varepsilon\to 0,$$

$$\frac{a_0+a_1u_{2,\varepsilon}}{v_{2,\varepsilon}}a_\varepsilon\to 0,\quad \frac{b_0+b_1v_{2,\varepsilon}}{u_{2.\varepsilon}}a_\varepsilon\to 0$$

And

$$ck_1+0\approx 0\qquad then\ \ k_1=0$$

The same for the second equation

$a_0 \longmapsto \bar{a}_0\,,\ a_1 \longmapsto \bar{a}_1\,,\ b_0 \longmapsto \bar{b}_0,\ b_1 \longmapsto, \overline{b_1}$

Then from the equation (5), we have

$$ck_2 + 0 = 0, \qquad then \quad k_2 = 0$$

And there are no SDW (both $k_1, k_2 = 0$)

Case 3: $u_{2,\varepsilon} \approx A_2,\ v_{1,\varepsilon} \approx B_1,\ u_{1,\varepsilon} \sim \frac{1}{\varepsilon},\ \ v_{2,\varepsilon} \sim \frac{1}{\varepsilon}$ (different sides are δ in u and v),

Then from (4), we have

$$\propto_0 = k_1,$$

And from (5) we have

$$\beta_1 = k_2$$

Thus the equation(4) implies that

$$c\,k_1 + \frac{b_1}{A_2}\beta_1 + \frac{a_1}{B_1}\propto_0 = 0 \qquad (9)$$

Since $\beta_1 = k_2,\ \propto_0 = k_1$

Then

$$(c + \frac{a_1}{B_1})k_1 + \frac{b_1}{A_2}k_2 = 0 \qquad (10)$$

Also the equation(5) implies that

$$ck_2 + \frac{\bar{a}_1}{B_1}\propto_0 + \frac{\bar{b}_1}{A_2}\beta_1 = 0$$

Then

$$\frac{\bar{a}_1}{B_1}k_1 + \left(c + \frac{\bar{b}_1}{A_2}\right)k_2 = 0 \qquad (11)$$

We can find solution for c (quadratic equations) from (10), $c = c_{1,2}$,

and from (11), $c = c_{3,4}$

Then condition for SDW is that one of these holds

$c_1 = c_3 or\ c_1 = c_4\ or c_2 = c_3\ or\ c_2 = c_4$

One has to check whether some for these solutions are physical (*u*, *v*≥ *0*, *etc*.)

Case 4: Only one of $u_{i,\varepsilon}$, $v_{i,\varepsilon}\ is\ \frac{1}{\varepsilon}$

Say that $u_{1,\varepsilon} \sim \frac{1}{\varepsilon}$, $u_{2,\varepsilon} \approx A_2$, $v_{1,\varepsilon} \approx B_1$, $v_{2,\varepsilon} \approx B_2$

Then from (4), we have

$$\propto_0 = k_1,$$

and from (5), we have

$$k_2 = 0$$

Thus that implies

$$c = \frac{\left[\frac{\bar{a}_0 + \bar{a}_1 u}{v} + \frac{\bar{b}_o + \bar{b}_1 v}{u}\right]}{[v]}$$

is determined

Then from (4), we have

$$ck_1 + \frac{a_1}{B_1} \propto_o = 0$$

since $\alpha_0 = k_1$

Then $c = -\frac{a_1}{B_1}$

Thus SDW exists if

$$B_1 = -\frac{[v]a_1}{\left[\frac{\bar{a}_o+\bar{a}_1 u}{v} + \frac{\bar{b}_o+\bar{b}_1 v}{u}\right]} \geq 0$$

If $u_{1,\varepsilon} \approx A_1$, $u_{2,\varepsilon} \sim \frac{1}{\varepsilon}$, $v_{1,\varepsilon} \approx B_1$, $v_{2,\varepsilon} \approx B_2$, then we get that

$$B_1 = B_2$$

Now, with the same argument, say $u_{1,\varepsilon} \approx A_1$, $u_{2,\varepsilon} \approx A_2$, $v_{1,\varepsilon} \sim \frac{1}{\varepsilon}$, $v_{2,\varepsilon} \approx B_2$

Then from (4), we have

$$\beta_0 = k_2,$$

and from (5), we have

$$k_1 = 0$$

Thus that implies

$$c = \frac{\left[\frac{a_0+a_1 u}{v} + \frac{b_0+b_1 v}{u}\right]}{[u]}$$

is determined.

Then from (4), we have

$$ck_2 + \frac{\bar{b}_1}{A_1}\beta_o = 0$$

since $\beta_0 = k_2$

Then $c = -\frac{\bar{b}_1}{A_1}$

Thus SDW exists if

$$A_1 = -\frac{[u]\bar{b}_1}{\left[\frac{a_0+a_1u}{v} + \frac{b_0+b_1\,v}{u}\right]} \geq 0$$

If $u_{1,\varepsilon} \approx A_1$, $u_{2,\varepsilon} \approx A_2$, $v_{1,\varepsilon} \sim \frac{1}{\varepsilon}$, $v_{2,\varepsilon} \approx B_2$, then we get that

$$A_1 = A_2.$$

4-Conclusion

We constructed solution to the system (1) using shadow wave solution with a split delta shock, In the cases 1,3and 4 there exist shadow wave solution. The solutions obtained by shadow wave with a split delta shock are overcompressive. The overcompressibility condition is not sufficient to case 2 because there is no shadow wave which is not satisfy admissibility criteria condition, the wave tends to zero as t tends to zero. Of course, there are a lot of specific situations. For a real model one has to check whether $(u_1,v_1) \in \Omega$ and an admissibility condition for split delta shocks, too.

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