حل المعادلات التفاضلية الجزئية باستخدام طريقة ادوميان المعدلة وتحويل عبعوب – شخيم

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الملخص:

نقدم في هذا العمل طريقة جديدة وهيا طريقة ادوميان المعدلة مع تحويل عبعوب – شخيم لحل المعادلات الجزئية ، تم اختبار هذه الطريقة لبعض الأمثلة وأظهرت النتائج موثيقية وكفاءة الطريقة المقترحة.

Solving the partial Differential Equations Using the Modified Abaoub-Shkheam Adomian Decomposition Method

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Abstract

In this work, we present a new method, the Adomian decomposition method with the Abaoub-Shkheam transformation for solving partial equations, test this method for some examples, and the results show the reliability and efficiency of the proposed method.

Keywords: Abaoub- Shkheam Transform- Adomian decomposition Method- partial differential equations.

1-introdaction

The Adomian decomposition method (ADM) introduced by Adomian [3 − 4] possesses a great potential in solving different kinds of functional equations. And can be used method to solve differential equations, both integer and fractional order, linear or nonlinear, ordinary or partial. This method can be combined with integral transformations, The Adomian decomposition methods can be combined with integral transformations, such as Laplace [17], Sumudu [18], Natural [19], Elzaki [20], Mohand [21], and Kashuri-Fundo [22].

Ali. Abaoub, and Abigail. Shkheam Abaoub- Shkheam recently introduced a new integral transform(QDM), dubbed the Abaoub- Shkheam transform in [1], and used it to solve ordinary and partial differential equations. Our purpose here is to show the applicability of this interesting new transformation and its effect on solving such problems. The technique of combining the Adomian decomposition method and the integral Abaoub-Shkheam transformation new shows how to be used to approximate the solutions of the nonlinear partial differential equations by manipulating the decomposition method. And will accelerate the rapid convergence of series solution when compared with the Laplace decomposition method and therefore provides major progress.

The Combined Method of the Adomian Decomposition Method and the Abaoub-Shkheam Integral Transformation is very accurate in solving partial differential equations and easier to determine the terms of the partial differential equation approximation solution the rest of the paper is organized as follows. the Abaoub-Shkheam integral transformation is presented in Section 2. The basic theory of the Modified Abaoub-Shkheam decomposition method in Section 3. In Section 4, the numerical solution of Adomian decomposition method and the partial differential integral transformation are demonstrated. The Conclusion is presented in Section 5.

2.Abaoub- Shkheam Transform "Q – Transform":

Definition2.1. Let $f(t)$ be a function defined for all $t \geq 0$, the Q-transform of $f(t)$ is the function $T(u,s)$ defined by

$$T(u,s) = Q[f] = \int_0^\infty f(ut)\, e^{-\frac{t}{s}}\, dt \qquad (2.1)$$

provided the integral exists for some s, where $s \in (-t_1, t_2)$.

The original function $f(t)$ in (1) is called the inverse transform or inverse of $T(u,s)$, and is denoted by $f(t) = Q^{-1}\{T(u,s)\}$.

If we substitute $ut = y$, then equation (1) becomes,

$$Q[f(t)] = T(u,s) = \frac{1}{u}\int_0^\infty f(y)\, e^{-\frac{1}{us}y}\, dy \qquad (2.2)$$

2.2. Abaoub-Shkheam transform for some basic functions

Elementary functions include algebraic and transcendental functions.

1. $Q[1] == s$
2. $Q[t^n] = n!\, u^n s^{n+1}, s > 0, n \in \mathbb{N} \cup \{0\}$

3. $Q\{\sin at\} = \frac{au\, s^2}{1+ a^2u^2s^2}$ where $\frac{1}{s} > au.$
4. $Q\{\cos at\} = \frac{s}{1+ a^2u^2s^2}$ where $\frac{1}{s} > au.$
5. $Q[e^{at}] = \frac{s}{1-a\, u\, s}$ where $\frac{1}{s} > au.$
6. $Q^{-1}\left\{ \frac{au\, s^2}{1+ a^2u^2s^2}\right\} = \sin at$.
7. $Q^{-1}\left\{\frac{s}{1+ a^2u^2s^2}\right\} = \cos at.$
8. $Q^{-1}\left\{\frac{s}{1+ a^2u^2s^2}\right\} = \cos at.$
9. $Q^{-1}\left\{\frac{s}{1-a\, u\, s}\right\} = e^{at}$

And Derivations the Abaoub- Shkheam Transform are

$$-Q\left[\frac{\partial f(x,t)}{\partial t}\right] = -\frac{1}{u}u(x,0) + \frac{1}{us}Q[u(x,t\,)] \qquad (2.3)$$

$$-Q\left[\frac{\partial^2 f(x,t)}{\partial t^2}\right] = \frac{Q[u(x,t\,)]}{u^2s^2} - \frac{1}{u^2s}u(x,0) - \frac{1}{u}u_t(x,0) \quad (2.4)$$

3.Modified Abaoub-Shkheam decomposition method

The purpose of this section is to discuss the use of modified Abaoub-Shkheam transform algorithm for the nonlinear partial differential equations. For convenience we consider the general form of second order nonhomogeneous nonlinear partial differential equations with initial conditions is given below

$$Lu(x,t) + Ru(x,t) + Nu(x,t) = hu(x,t) \qquad (3.1)$$

$$u(x,0) = f(x), \quad u_t(x,0) = g(x) \qquad (3.2)$$

where L is second order differential operator $L = \frac{\partial^2}{\partial t^2}$, R is the is remaining linear operator, Nu represents a general non-linear differential operator and $h(x,t)$ is source term. The methodology consists of applying Abaoub-Shkheam transform first on both sides of Eq. (3.1).

$$Q[Lu(x,t)] + Q[Ru(x,t)] + Q[Nu(x,t)] = Q[hu(x,t)] \qquad (3.3)$$

Using the differentiation property of Abaoub-Shkheam transform we get

$$\frac{Q[u(x,t)]}{u^2s^2} - \frac{1}{u^2s}f(x) - \frac{1}{\mathrm{u}}\, g(x) + Q[Ru(x,t)] + Q[Nu(x,t)] = Q[hu(x,t)] \qquad (3.4)$$

$$Q[u(x,t)] = sf(x) + \mathrm{u}s^2\, g(x) - u^2s^2Q[Ru(x,t)] - u^2s^2Q[Nu(x,t)] + u^2s^2Q[hu(x,t)] \quad (3.5)$$

The second step in Abaoub-Shkheam decomposition method is that we represent solution
as an infinite series given below

$$u = \sum_{n=0}^{\infty} u_n(x,t) \qquad (3.6)$$

The nonlinear operator is decompose as

$$Nu(x,t)] = \sum_{n=0}^{\infty} A_n \qquad (3.7)$$

Where A_n are Adomian polynomials [10] of $u_0, u_1, u_2, \ldots, u_n$ and it can be calculated by formula given below

$$A_n = \frac{1}{n!}\frac{d^n}{dx^n}\left[N\left(\sum_{i=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0}, \quad n = 0,1,2,\ldots \qquad (3.8)$$

Using Eq. (3.6) and Eq. (3.7) in Eq. (3.5) we will get

$$Q\left[\sum_{n=0}^{\infty} u_n(x,t)\right] = sf(x) + \mathrm{u}s^2\, g(x) - u^2s^2Q[Ru(x,t)] - u^2s^2Q\left[\sum_{n=0}^{\infty} A_n\right] + u^2s^2Q[hu(x,t)] \quad (3.9)$$

$$\sum_{n=0}^{\infty} Q[u_n(x,t)] = sf(x) + \mathrm{u}s^2\, g(x) - u^2s^2Q[Ru(x,t)] - u^2s^2Q\left[\sum_{n=0}^{\infty} A_n\right] + u^2s^2Q[hu(x,t)] \quad (3.10)$$

On comparing both sides of the Eq. (3.10) we have

$$Q\big(u_0(x,t)\big) = sf(x) + \mathrm{u}s^2\, g(x) + u^2s^2Q[hu(x,t)] = k(x,s) \quad (3.11)$$

$$Q\big(u_1(x,t)\big) = -u^2s^2Q[Ru_0(x,t)] - u^2s^2Q[A_0] \quad (3.12)$$

$$Q\big(u_2(x,t)\big) = -u^2s^2Q[Ru_1(x,t)] - u^2s^2Q[A_1] \quad (3.13)$$

In general, the recursive relation is given by

$$Q\big(u_{n+1}(x,t)\big) = -u^2s^2Q[Ru_n(x,t)] - u^2s^2Q[A_n] \quad ,n \geq 1 \quad (3.14)$$

Applying inverse Abaoub-Shkheam transform to Eq. $(3.11) - (3.14)$, so our required

recursive relation is given below

$$u_0(x,t) = k(x,t) \quad (3.15)$$

$$u_{n+1}(x,t) = -Q^{-1}(u^2s^2Q[Ru_n(x,t)] + u^2s^2Q[A_n]\,) \quad ,n \geq 1 \quad (3.16)$$

where $k(x,t)$represent the term arising from source term and prescribe initial conditions. Now first of all we applying Abaoub-Shkheam transform of the terms on the right hand side of Eq. (3.16) then applying inverse Abaoub-Shkheam transform we get the values of $u_0, u_1, u_2, \dots, u_n$ respectively.

To apply this modification, we assume that $k(x,t)$can be divided into the sum of two parts namely$k_0(x,t)$ and $k_1(x,t)$, therefore we get

$$k(x,t) = k_0(x,t) + k_1(x,t) \quad (3.17)$$

Under this assumption, we propose a slight variation only in the components u_0, u_1. The variation we propose is that only the part $k_0(x,t)$be assigned to the u_0,whereas the remaining part $k_1(x,t)$be combined with the other terms given in Eq. (3.16) to defineu_1. In view of these suggestion, we formulate the modified recursive algorithm as follows:

$$u_0(x,t) = k_0(x,t) \quad (3.18)$$

$$u_1(x,t) = k_1(x,t) - u^2s^2Q[Ru_0(x,t)] - u^2s^2Q[A_0] \quad (3.19)$$

$$Q\big(u_{n+1}(x,t)\big) = -Q^{-1}(u^2s^2Q[Ru_n(x,t)] + u^2s^2Q[A_n]\,) \quad ,n \geq 1 \quad (3.20)$$

The solution through the modified Adomian decomposition method is highly depend upon the choice of $k_0(x,t)$ and $k_1(x,t)$.

4. Applications

To illustrate this method for nonlinear partial differential equations we take three examples in this section.

4.1 Example 1

Consider a nonlinear partial differential equation [7]

$$\frac{\partial u(x,t)}{\partial t} + uu_x = x + xt^2 \quad (4.1)$$

with initial conditions

$$u(x,0) = 0 \quad (4.2)$$

Applying the Abaoub-Shkheam transform (denoted by Q) we have

$$\frac{1}{us}Q[u(x,t)]-\frac{1}{u}u(x,0)= Q[x+xt^2]-Q[uu_x] \quad (4.3)$$

Using initial conditions Eq. (4.2) becomes

$$Q[u(x,t)]=us^2x+2!\,xu^3s^4-usQ[uu_x] \quad (4.4)$$

Applying inverse Abaoub-Shkheam transform we get

$$u(x,t)=xt+\frac{xt^3}{3}-Q^{-1}(usQ[uu_x]\,) \quad (4.5)$$

We decompose the solution as an infinite sum given below

$$u=\sum_{n=0}^{\infty}u_n(x,t) \qquad (4.6)$$

The nonlinear term is handled with the help of Adomian polynomials [10] given below

$$uu_x=\sum_{n=0}^{\infty}A_n(u) \quad (4.7)$$

Using Eq. $(4.6)-(4.7)$ in equation (4.5) we get

$$\sum_{n=0}^{\infty}u_n(x,t)\ =xt+\frac{xt^3}{3}-Q^{-1}(usQ\left[\sum_{n=0}^{\infty}A_n(u)\right]) \quad (4.8)$$

The recursive relation is given below

$$u_0(x,t)=xt+\frac{xt^3}{3} \qquad (4.9)$$

$$u_1(x,t)=-Q^{-1}(usQ[A_0(u)])=-Q^{-1}(usQ[u_0u_{0x}])$$

$$=-Q^{-1}(usQ\left[\left(xt+\frac{xt^3}{3}\right)\left(t+\frac{t^3}{3}\right)\right])$$

$$=-Q^{-1}(usQ\left[xt^2+\frac{2xt^4}{3}+\frac{xt^6}{9}\right])$$

$$=-Q^{-1}(us\left[2xu^2s^3+\frac{2x}{3}4!\,u^4s^5+\frac{x}{9}6!\,u^6s^7\right])$$

$$=-Q^{-1}(\left[2xu^3s^4+\frac{2x}{3}4!\,u^5s^6+\frac{x}{9}6!\,u^7s^8\right])$$

$$u_1(x,t)=\frac{-x}{3}t^3-\frac{2x}{15}t^5-\frac{x}{63}t^7 \qquad (4.10)$$

$$u_2(x,t)=-Q^{-1}(usQ[A_1(u)])=-Q^{-1}(usQ[u_1u_{0x}+u_0u_{1x}])$$

$$= -Q^{-1}(usQ\left[\left(\frac{-x}{3}t^3 - \frac{2x}{15}t^5 - \frac{x}{63}t^7\right)\left(t + \frac{t^3}{3}\right) + \left(xt + \frac{xt^3}{3}\right)\left(\frac{-x}{3}t^3 - \frac{2x}{15}t^5 - \frac{x}{63}t^7\right)\right])$$

$$= Q^{-1}(usQ\left[\frac{-2xt^4}{3} - \frac{2xt^6}{9} - \frac{2xt^8}{63} - \frac{2xt^{10}}{198} - \frac{4xt^6}{15} - \frac{4xt^8}{45}\right])$$

$$= Q^{-1}(us\left[\frac{-2x}{3}4!\,u^4s^5 - \frac{2x}{9}6!\,u^6s^7 - \frac{2x}{63}8!\,u^8s^9 - \frac{2x}{198}10!\,u^{10}s^{11} - \frac{4x}{15}6!\,u^6s^7 - \frac{4x}{45}8!\,u^8s^9\right])$$

$$= Q^{-1}(\frac{-2x}{3}4!\,u^5s^6 - \frac{2x}{9}6!\,u^7s^8 - \frac{2x}{63}8!\,u^9s^{10} - \frac{2x}{198}10!\,u^{11}s^{12} - \frac{4x}{15}6!\,u^7s^8 - \frac{4x}{45}8!\,u^9s^{10})$$

$$u_2(x,t) = \frac{2x}{15}t^5 + \frac{22x}{315}t^7 + \frac{38}{2835}t^9 + \frac{2x}{2079}t^{11} \quad (4.11)$$

$$u_{n+1}(x,t) = -Q^{-1}\left(usQ\left[\sum_{n=0}^{\infty} A_n(u)\right]\right) \quad ,n \geq 1 \quad (4.12)$$

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = xt \quad (4.13)$$

We consider same nonlinear partial differential equation [7] and solve it by using modified Abaoub-Shkheam decomposition method

$$\frac{\partial u(x,t)}{\partial t} + uu_x = x + xt^2 \quad (4.14)$$

with initial conditions

$$u(x,0) = 0 \quad (4.15)$$

Applying the Abaoub-Shkheam transform we have

$$\frac{1}{us}Q[u(x,t)] - \frac{1}{u}v(x,0) = Q[x + xt^2] - Q[uu_x] \quad (4.16)$$

Using initial conditions Eq. (4.16) becomes

$$Q[u(x,t)] = us^2x + 2!\,xu^3s^4 - usQ[uu_x] \quad (4.17)$$

Applying inverse Abaoub-Shkheam transform we get

$$u(x,t) = xt + \frac{xt^3}{3} - Q^{-1}(usQ[uu_x]) \quad (4.18)$$

We decompose the solution as an infinite sum given below

$$u = \sum_{n=0}^{\infty} u_n(x,t) \qquad (4.19)$$

The nonlinear term is handled with the help of Adomian polynomials [10] given Below

$$uu_x = \sum_{n=0}^{\infty} A_n(u) \qquad (4.20)$$

Using Eq. $(4.19) - (4.20)$ in equation (4.18) we get

$$\sum_{n=0}^{\infty} u_n(x,t) = xt + \frac{xt^3}{3} - Q^{-1}\left(usQ\left[\sum_{n=0}^{\infty} A_n(u)\right]\right)$$

The recursive relation is given below

$$u_0(x,t) = xt \quad (4.21)$$

$$u_1(x,t) = \frac{xt^3}{3} - Q^{-1}(usQ[A_0(u)])$$

$$u_{n+1}(x,t) = -Q^{-1}(usQ[A_n(u)]) \qquad (4.22)$$

where$A_n(u)$ are Adomian polynomials representing the nonlinear terms [10] in above Eq. (4.22)In view of the recursive relations $(4.21) - (4.22)$ we obtained other components as follows

$$u_1(x,t) = \frac{xt^3}{3} - Q^{-1}(usQ[A_0(u)]) = -Q^{-1}(usQ[u_0 u_{0x}])$$

$$= \frac{xt^3}{3} - Q^{-1}(usQ[xt(t)])$$

$$= \frac{xt^3}{3} - Q^{-1}(us[2xu^2s^3])$$

$$= \frac{xt^3}{3} - Q^{-1}(2xu^3s^4)$$

$$= \frac{xt^3}{3} - \frac{x}{3}t^3$$

$$= 0,$$

$$u_1(x,t) = 0 \qquad (4.23)$$

$$u_{n+1}(x,t) = 0 \qquad (4.24)$$

In view of above modified recursive relation we get exact solution

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = xt \qquad (4.25)$$

4.2 Example 2

Consider another nonlinear partial differential equation [7]

$$\frac{\partial^2 u(x,t)}{\partial x^2} - u_x u_{yy} = -x + u \qquad (4.26)$$

with initial conditions

$$u(x,0) = siny \ , \ u_x(x,0) = 1 \qquad (4.27)$$

Applying the Abaoub-Shkheam transform we have

$$\frac{1}{u^2 s^2} Q[u(s,y)] - \frac{1}{u^2 s} u(y,0) - \frac{1}{\mathrm{u}} u_x(y,0)$$
$$= Q[-x+u] + Q[u_x u_{yy}] \qquad (4.28)$$

Using initial conditions Eq. (4.28) becomes

$$Q[u(s,y)] = ssiny + \mathrm{us}^2 - u^3 s^4 + u^2 s^2 Q[u] + u^2 s^2 Q[u_x u_{yy}] \qquad (4.29)$$

Applying inverse Abaoub-Shkheam transform we get

$$u(x,y) = siny + x - \frac{x^3}{3!} + Q^{-1}(u^2 s^2 Q[u + u_x u_{yy}]) \qquad (4.30)$$

Applying the same procedure as in previous example we arrive at modified recursive relation given below

$$u_0(x,y) = siny + x \qquad (4.34)$$

$$u_1(x,y) = -\frac{x^3}{3!} + Q^{-1}\left(u^2 s^2 Q\left[\sum_{n=0}^{\infty} u_n(x,t) + \sum_{n=0}^{\infty} B_0(u)\right]\right) \qquad (4.35)$$

$$u_{n+1}(x,y) = Q^{-1}(u^2 s^2 Q[u_n + \textstyle\sum_{n=0}^{\infty} B_n(u)]) , n \geq 1 \qquad (4.36)$$

where $B_n(u)$is a Adomian polynomials [10] representing the nonlinear terms in above Eq. (4.36),. The other components of the series solution can be calculated by using above recursive relation

$$u_0(x,t) = siny + ux \qquad (4.37)$$

$$u_1(x,y) = -\frac{x^3}{3!} + Q^{-1}\left(u^2 s^2 Q\left[u_0 + \sum_{n=0}^{\infty} B_0(u)\right]\right) \qquad (4.38)$$

$$= -\frac{x^3}{3!} + Q^{-1}(u^2 s^2 Q[u_0 + u_{0x} u_{yy}])$$

$$= -\frac{x^3}{3!} + Q^{-1}(u^2 s^2 Q[siny + x - siny])$$
$$= -\frac{x^3}{3!} + Q^{-1}(u^2 s^2 Q[x])$$
$$= -\frac{x^3}{3!} + Q^{-1}(u^2 s^2 [us^2])$$
$$= -\frac{x^3}{3!} + Q^{-1}(u^3 s^4)$$
$$= -\frac{x^3}{3!} + \frac{x^3}{3!}$$
$$= 0,$$
$$u_1(x,y) = 0 \qquad (4.39)$$
$$u_{n+1}(x,t) = 0 \quad ,n \geq 1 \quad (4.40)$$

The total solution of the above problem is given below

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \ = x + siny \qquad (4.41)$$

4.3 Example 3

We consider system of coupled nonlinear partial differentials [7]

$$\frac{\partial u(x,t)}{\partial t} + wu_x + u = 1 \quad ,t > 0 \ (4.42)$$
$$\frac{\partial w(x,t)}{\partial t} - uw_x - w = 1 \qquad (4.43)$$

with initial conditions

$$u(x,0) = e^x \ (4.44)$$
$$w(x,0) = e^{-x} \qquad (4.45)$$

To solve Eqs. (4.42) $-$ (4.43), first of all we will apply Abaoub-Shkheam transform and using given initial conditions we get

$$\frac{1}{us}Q[u(x,t)] - \frac{1}{u}u(x,0) = \ Q[1 - wu_x - u] \quad (4.46)$$
$$\frac{1}{us}Q[w(x,t)] - \frac{1}{u}w(x,0) = \ Q[1 + uw_x + w] \quad (4.47)$$

Using initial conditions Eqs. (3.46) $-$ (3.47) becomes

$$Q[u(x,t)] = se^x + us^2 - usQ[wu_x + u] \quad (4.48)$$
$$Q[w(x,t)] = se^{-x} + us^2 + usQ[uw_x + w] \quad (4.49)$$

Applying inverse Abaoub-Shkheam transform we get

$$u(x,t) = e^x + t - Q^{-1}(usQ[wu_x + u]) \quad (4.50)$$
$$w(x,t) = e^{-x} + t + Q^{-1}(usQ[uw_x + w]) \quad (4.51)$$

We decompose the solution as an infinite sum given below

$$u = \sum_{n=0}^{\infty} u_n(x,t) \quad (4.52)$$
$$w = \sum_{n=0}^{\infty} w_n(x,t) \quad (4.53)$$

Using Eq. (4.52) − (4.53) we have

$$\sum_{n=0}^{\infty} u_n(x,t) = e^x + t - Q^{-1}\left(usQ\left[\sum_{n=0}^{\infty} C_n(x,t) + \sum_{n=0}^{\infty} u_n(x,t)\right]\right) \quad (4.54)$$

$$\sum_{n=0}^{\infty} w_n(x,t) = e^{-x} + t + Q^{-1}\left(usQ\left[\sum_{n=0}^{\infty} D_n(x,t) + \sum_{n=0}^{\infty} w_n(x,t)\right]\right) \quad (4.55)$$

where $C_n(x,t), D_n(x,t)$ are Adomian polynomials [10]. Now our required modified recursive relations are given by

$$u_{n+1}(x,t) = -Q^{-1}(usQ[C_n(u,w) + u_n]) \quad , n \geq 1 \quad (4.56)$$

$$w_{n+1}(x,t) = Q^{-1}(usQ[D_n(u,w) + w_n]) \quad , n \geq 1 \quad (4.57)$$

Where$C_n(x,t), D_n(x,t)$are Adomian polynomials representing nonlinear terms [10]. Applying the same procedure as describe in previous example, we get

$$u_0(x,t) = e^x \quad (4.58)$$

$$u_1(x,t) = t - Q^{-1}(usQ[C_0(u,w) + u_0]) = t - Q^{-1}(usQ[w_0 u_{0x} + u_0])$$
$$= t - Q^{-1}(usQ[e^{-x}e^x + e^x])$$
$$= t - Q^{-1}(usQ[1 + e^x])$$
$$= t - Q^{-1}(us[s + se^x])$$
$$= t - Q^{-1}(us^2 + us^2 e^x)$$
$$= t - t - te^x$$

$$u_1(x,t) = -te^{x} \quad (4.59)$$

$$u_2(x,t) = -Q^{-1}(usQ[C_1(u,w) + u_1])$$

$$= -Q^{-1}(usQ[w_1 u_{0x} + w_0 u_{1x} + u_1])$$

$$= -Q^{-1}(usQ[te^{-x}e^{x} + e^{-x}(-te^{x}) - te^{x}])$$

$$= -Q^{-1}(usQ[-te^{x}])$$

$$= -Q^{-1}(us[-us^2 e^{x}])$$

$$= -Q^{-1}(-u^2 s^3 e^{x})$$

$$u_2(x,t) = \frac{t^2}{2!}e^{x} \quad (4.60)$$

$$w_0(x,t) = e^{-x} \quad (4.61)$$

$$w_1(x,t) = t + Q^{-1}(usQ[D_0(u,w) + w_0]) = t - Q^{-1}(usQ[u_0 w_{0x} + w_0])$$

$$= t + Q^{-1}(usQ[e^{x}(-e^{-x}) + e^{-x}])$$

$$= t + Q^{-1}(usQ[-1 + e^{-x}])$$

$$= t + Q^{-1}(us[-s + se^{-x}])$$

$$= t + Q^{-1}(-us^2 + us^2 e^{-x})$$

$$= t - t + te^{-x}$$

$$w_1(x,t) = te^{-x} \quad (4.61)$$

$$w_2(x,t) = Q^{-1}(usQ[D_1(u,w) + w_1])$$

$$= -Q^{-1}(usQ[u_1 w_{0x} + u_0 w_{1x} + w_1])$$

$$= Q^{-1}(usQ[-te^{x}(-e^{x}) + e^{x}(-te^{-x}) + te^{-x}])$$

$$= Q^{-1}(usQ[te^{-x}])$$

$$= Q^{-1}(us[us^2 e^{-x}])$$

$$= Q^{-1}(u^2 s^3 e^{-x})$$

$$w_2(x,t) = \frac{t^2}{2!}e^{x} \quad (4.62)$$

In view of Eqs $(4.56) - (4.62)$,the series solution is

$$u(x,t) = e^{x} - te^{x} + \frac{t^2}{2!}e^{x} - \cdots$$

$$= e^{x}[1 - t + \frac{t^2}{2!} - \cdots$$

$$u(x,t) = e^{x-t} \quad (4.63)$$

$$w(x,t) = e^{-x} + te^{-x} + \frac{t^2}{2!}e^{x} + \cdots$$

$$= e^{-x}[1 + t + \frac{t^2}{2!} + \cdots$$

$$w(x,t) = e^{t-x} \quad (4.64)$$

With the help of this modification we are able to get exact solution very easily.

4 Conclusion

In this paper, we carefully presented a dependable adjustment to the Abaoub-Shkheam decomposition approach Three nonlinear partial differential equations with beginning conditions were solved. Example 1 was solved in two ways: first using the Abaoub-Shkheam decomposition approach, and then using a modified Abaoub-Shkheam decomposition method. This example demonstrates the presence of "noise terms" in the components of the solution series generated using the Abaoub-Shkheam decomposition technique. While the solution produced by modified Abaoub-Shkheam decomposition does not contain any noise components. Examples 2 and 3, which are nonlinear coupled partial differential equations, are solved in a similar manner. This modified technique has been shown to computationally efficient in these examples that are important to researchers in the field of applied sciences. In addition, the modified Abaoub-Shkheam decomposition method may give the exact solutions for nonlinear partial or coupled partial differential equations.

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